Mueller matrix imaging polarimetry for the characterization of turbid medium

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ABSTRACT

The Mueller matrix polarimetry is a useful technique for characterization and imaging of biological materials and turbid medium. The recoded transmitted Mueller matrix elements are highly intensity and wavelength dependent. We derived the sixteen Mueller matrix images out of forty-nine polarization configurations for characterization of the polystyrene sphere of 0.2 to 20 µm illuminated with polarized He-Ne laser beam. The measured Mueller matrix along with 3D and 2D images provides a complete fingerprint of the optical properties and activity of turbid medium. Polarization imaging technique is more powerful then video reflectrometry or other imaging techniques. We presented the system for experimental measurements of Mueller matrix elements and depths resolve transmission images for characterization of scattering and depolarization of birefrengent polystyrene sphere sample.

KEYWORDS: `optical imaging, polarization, scattering, depolarization, turbid medium, Mueller matrix`

In recent years, there has been increased interest in using non harmful light for studying and diagnostic human pathology because of the noninvasive nature of the light tissue interaction [1]. Polarimetry is a powerful tool that has been employed in various disciplines [2]. A few recent studies have demonstrated that information on the properties of a turbid medium can be measured by shining a polarized laser beam onto a sample and then analyzing the state of polarization of the transmitted light. The polarization of light has been used to extract quantitative information from the optically thick medium with which it interacts. Indeed, although the degree of polarization is significantly lowered after propagating through thick turbid media, often it is not completely destroyed [3]. Polarization is well suited for measuring birefringence and optical activity, which affect, respectively for the ellipticity and the orientation of the polarization of the incident beam. For instance, birefringence measurement [4]. The investigated applications of this technique include the measurements of the average particle size, scattering coefficients and anisotropy factor of particle suspensions [5], the study of biological materials [6-7], measurements of the average photon path length [8], polarization-sensitive optical coherence tomography OCT measurements [9,10], and imaging of polarization-sensitive skin pathology [11]. The benefits associated with use of polarization information lidar multiple scattering from atmospheric clouds [12], and Mueller matrix imaging for underwater target detection in turbid media [13] are the topic of resent interest. In order to achieve full experimental characterization of the optical properties of the sample under investigation, we applied the concept of an effective Mueller matrix [14-17], and used transmitted polarized light from highly scattering media, including measurements of linear and circularly polarized light. Several research groups are thus investigating the response of turbid media to polarized light by examining the polarization properties of multiply scattered light [18,19]. These polarization methodologies have been developed to study various aspects of polarized light interaction with turbid media, including the investigation of the depolarization mechanisms, determination of system Mueller matrices of scattered light and polarization imaging studies of multiply scattering media .

We present an experimental framework involving use of the Mueller matrix to investigate polystyrene sphere suspension in deionized water. The Mueller matrix approach is explored to improve use of polarimetric scattering. The Mueller matrix is an important parameter in the study of the polarization configuration associated with light scattering and radiative transfer processes. In practice, the 16 Mueller matrix elements can be determined by taking 49 polarimetric output analyzer polarization states [20]. We investigate the sensitivity of the contours of the scattering Mueller matrix to particle shape following the theoretical framework associated with light and using well-known characteristic of optical activity for polystyrene sphere solution to rotate the plane of linearly polarized light about the axis of propagation. The amount of rotation depends on the molecular concentration, the path length through the medium and the optical rotator power at the interrogation wavelength [21]. In this paper, we analyze the polarization characteristics of polystyrene sphere suspension through polarized transmitted light for imaging the medium and develop a method to explicitly derive linear and circular polarization image informations.

BACKGROUNDS

In general, the interaction of light with optical elements such as lenses, polarizer, filters, surfaces, scattering media etc., changes the polarization state of the light. When light is described by a four-component vector, this interaction with any optical element or material can be described as a multiplication of the Stokes vector with a 4x4 matrix, this sixteen-element matrix is called the Mueller matrix [22] if scattering is involved, this matrix completely characterizes any component or material in terms of its optical properties. For the light propagating along the z axis, we obtain the two orthogonal complex electric field components as:

$$
\begin{aligned}
E_x(t) &= E_{ix} e^{i\varphi_x(t)}\, e^{-iKz} e^{i\omega t}, \\
E_y(t) &= E_{iy} e^{i\varphi_y(t)}\, e^{-iKz} e^{i\omega t}
\end{aligned}
\qquad (1)
$$

where the phases $\varphi_x(t)$, $\varphi_y(t)$ and amplitudes $E_{ix}(t)$ & $E_{iy}(t)$ of each component are functions of time. The Stokes parameters, which are used to describe the polarization state of the light, are given in the vector form by [22]

$$S=\begin{bmatrix}S_0\\S_1\\S_2\\S_3\end{bmatrix}=\begin{bmatrix}\langle E_{0x}^2\rangle+\langle E_{0y}^2\rangle\\ \langle E_{0x}^2\rangle-\langle E_{0y}^2\rangle\\ 2\langle E_{0x}E_{0y}\cos\delta\rangle\\ 2\langle E_{0x}E_{0y}\sin\delta\rangle\end{bmatrix} \qquad (2)$$

where S_0 is total input intensity, S_1 and S_2 represent the linearly polarized components of the beam, S_3 represents the circularly polarized component and $\beta(t)=\varphi_x(t)+\varphi_y(t)$ is the relative phase difference between the two orthogonal components. For an arbitrary light beam, these terms are related by $S_0^2 \geq S_1^2+S_2^2+S_3^2$, the equality holds for fully polarized light and the inequality applies to partially polarized light. The degree of polarization for the turbid medium is defined as [23]

$$DOP=\sqrt{\frac{S_1^{\,2}+S_2^{\,2}+S_3^{\,2}}{S_0^{\,2}}} \qquad (3)$$

The transformation of the Stokes vector by an optical system is given by [24]

$$[S_{out}]=[M_{system}][S_{in}] \qquad (4)$$

where $[M_{system}]$ is the Mueller matrix representing the entire experimental optical system given as

$$[M_{system}]=[QW][A_M][M][QW][P_M] \qquad (5)$$

To model the polarization effects of various optical components were represented by Mueller matrices. Using Mueller calculus, an optical element that acts on a light beam is represented by multiplication of the incident light Stokes vector by the Mueller matrix for that optical element. The output stokes vector can be calculated by relation in eq.5. Where the Mueller matrix for polarizer $[P]$, turbid medium $[M]$, analyzer $[A]$ quarter wave plat $[QW]$ at horizontal fast axis and Stokes input vector $[S_{in}]$ are given as [25]

$$[P_M]=\frac{1}{2}\begin{bmatrix}1 & C_{2i} & S_{2i} & 0\\ C_{2i} & C_{2i}^2 & \frac{S_{4i}}{2} & 0\\ S_{2i} & \frac{S_{4i}}{2} & S_{2i}^2 & 0\\ 0 & 0 & 0 & 0\end{bmatrix};\quad [A_M]=\frac{1}{2}\begin{bmatrix}1 & C_{2o} & S_{2o} & 0\\ C_{2o} & C_{2o}^2 & \frac{S_{4o}}{2} & 0\\ S_{2o} & \frac{S_{4o}}{2} & S_{2o}^2 & 0\\ 0 & 0 & 0 & 0\end{bmatrix};$$

$$[QW]=\frac{1}{2}\begin{bmatrix}1 & -1 & 0 & 0\\ -1 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}\qquad [M]=\begin{bmatrix}m_{11} & m_{12} & m_{13} & m_{14}\\ m_{21} & m_{22} & m_{23} & m_{24}\\ m_{31} & m_{32} & m_{33} & m_{34}\\ m_{41} & m_{42} & m_{43} & m_{44}\end{bmatrix};$$

$$\text{and}\quad S_{in}=\begin{bmatrix}S_0\\S_1\\S_2\\S_3\end{bmatrix} \qquad (6)$$

where $C_{2i}=\cos(2\theta_i)$, $S_{2i}=\sin(2\theta_i)$, $S_{4i}=\sin(4\theta_i)$, $C_{2o}=\cos(2\theta_o)$, $S_{2o}=\sin(2\theta_o)$, $S_{4o}=\sin(4\theta_o)$. All the Mueller matrix elements can be determined experimentally [see *Fig.1*]. It can be shown that 49 intensity measurements with various orientations of polarizers and analyzers are necessary to obtain the 16 elements of the Mueller matrix [26]. *Tab.1* lists the necessary measurements for each matrix element [27]. Once all 16 elements of the matrix are obtained, the medium is completely described in terms of its optical characteristics [28]. The sample is of polystyrene-sphere of diameters 0.2 to 20 um and were suspended in deionized water. Approximately 50 ml of suspension in a 4x 2.5 cm cylindrical beaker was used for the scattering experiments. The probing light may be linearly polarized at various angles, right-hand circularly and left-hand circularly polarized. The necessary procedure is demonstrated by the Stokes-vector Mueller matrix approach of polarization imaging [29-30].

RESULTS

The objective of constructing the Mueller matrix imaging polarimeter is to measure the characteristics of polystyrene turbid medium from the Mueller matrix distribution. The images of the Mueller matrix elements of incident light of different polarizations states are shown in *FIg.2,3*. The image size is 1.3 cm physical and 1.2 cm optical for the 1.4 index of refraction of the medium. The physical depth of the images can be taken by dividing optical depth of the images with refractive index of investigated material. The M_{11} element of the Mueller matrix corresponds to the polarization independent image, as acquired through non polarized configuration. The other matries M_{ij}, (i and j=1,2,3,4) are normalized through M_{11}. The M11(total intensity matrix) reveals less informations then other strong layered structure, like M_{13}, M_{22}, M_{33}, and M_{44} we have taken multi images and readings of different polarizations states for polystyrene sphere suspension vibrating in current equsition time. All the other elements except M_{11} (identity matrix) can be negative due to intensities addition and subtraction, according to *Tab.1*. Looking at *Fig.2,3*, some Mueller matrix elements and images are symmetric ($M_{ij}=M_{ji}$) and other can simply attain by varying the Mueller matrix through some angles. All the Mueller matrix of polystyrene sphere shows azimuthally variation in relative intensity, and the intensity decay and increase depend upon scattering and absorption of sample. Locking at three dimensional images in *Fig.3*, one can see that the central images region of the sample have different optical polarization from those of the surrounding one, and the change in the state of polarization of transmitted scattered light from this region is attributed to optical birefringence and scattering. The sixteen Mueller matrix measurements by the method given in Eq.4 the output images are compensated with polarization optics and noted in the form of matrix [see *Fig.5*]. Some regions of the Mueller matrix images have strong cross polarized signals, which shows a maximum intensity profile in the three dimensional images of *Fig.3*. So the incident light is partially converted to the cross polarizations states by the clusters in those regions. The transmitted light in most of the regions preserves original polarizations states due to stronger co polarized signal instead of the cross polarized. This characterization technique has advantages over standard video reflectrometry (based on unpolarized light measurements). The entire matrix element provides comprehensive and detailed information of the sample. Analyzing only the diagonal elements or images M_{11}, M_{22}, M_{33}, and M_{44} of the Mueller matrix, one can extract useful information's and characterize the sample under investigation. M_{11} is unity and unpolarized matrix provides less information comparable to others but the maximum intensity pattern is achieved through this matrix. M_{22} and M_{33} shows the linear polarization effect of light through sample. M_{44} involves on circular polarized light only and has very less azimuthally dependence, it decrease with increasing sphere size as most of light reflected through the sample. The increase in the polystyrene sphere suspension concentration leads to an increase in the random scattering that the light undergoes per unit optical depth. An increase in the optical depth means that the scattered light undergoes more scattering events, which causes more fluctuation because each scattering event has Brownian motion. Therefore the average intensity increases with both the optical depth and the scatterer concentration in turbid medium, which would accordingly decrease the apparent DOP. This supposition can ultimately be tested if our setup is improved such that the Stokes vector of a sample can be measured in a sufficiently short time period [see *Fig.5*]. In the centre of Mueller matrix the output images shows strong polarization effect, due to less scattering of light. M_{14} is approximately equal to M_{41} regardless of the particles aspect ratio. The symmetry pattern of the Mueller matrix element is very dominant and the only sizes of the numerical value of the matrix are opposite. M_{22} and M_{33} are quite sensitive to the polystyrene sphere aspect ratios and may be useful for determining particles morphology. The Mueller matrix of upper left corner is associated only with the linear polarization configuration and easer to quantify by lab measurements then the circular polarization. This contour of the Mueller matrix are similar to those shown by Rakovic et all [13], who compared the experimental and theoretical results for the Mueller matrix associated with polystyrene sphere.

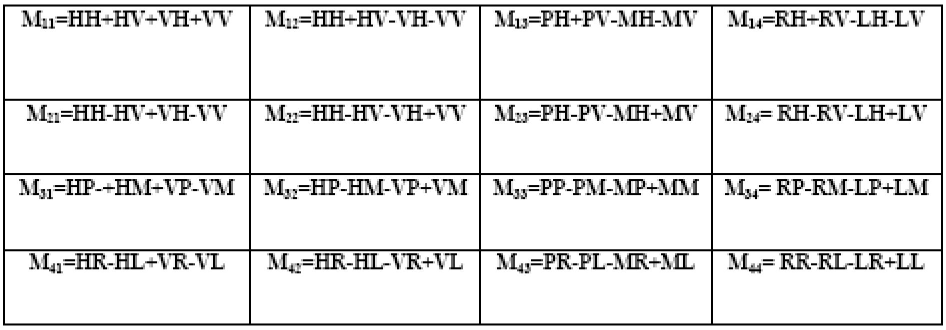
The results in *Fig.2,3* show good agreement between theory and experiment for the intensity pattern of three and two dimensional transmitted light. This technique can extract or decode useful informations about the sample under investigation.

CONCLUSION

For the determination of the sixteen Mueller matrix elements a total of 49 images are taken at various combination of input and output polarization states and all the sixteen out put elements are calculated by adding or subtracting a series of images. All the sixteen Mueller matrix components give a detailed and comprehensive knowledge about the birefrengent medium. It provides the change in media and need detailed analysis of each single Mueller matrix. The introduction of the Mueller-matrix concept for the transmitted light provides, in addition to the widely used M_{11} element and 15 more elements, which can be evaluated to obtain further information about scattering media. Because the description of the optical characteristics with the Mueller matrix is complete, any information about particle size, refractive index, particle shape etc. has to be found in the Mueller matrix by careful analysis of the matrix elements. If some information about the medium cannot be extracted from the various matrix elements, this information cannot be extracted from any other additional scattering measurements. However, further information may be gained, for example, by measuring the diffuse backscattering, the reflectance at different incident and observation angles, or time-dependent polarization effects. In this work we present a method and general framework for the study of transmitted polarized light from highly scattering media. Through this transmitted light, complex spatially varying surface-intensity patterns can be observed, and can be used to gain additional informations about the scattering medium. The Mueller matrix measurements using polarized light can be performed by varying the polarization state of the incident beam and detecting different polarization components of the transmitted light. However, by introducing the Stokes-vector, Mueller-matrix concept, we comprehensively describes the optical properties of birefrengent media. Further studies are necessary to fully explore the Mueller-matrix approach for characterization of turbid medium and its possible use in biomedical diagnostics and treatment procedures.

ACKNOWLEDGMENTS

The authors acknowledge the enabling role of the Higher Education Commission Islamabad, Pakistan and appreciate its financial support through “Merit Scholarship Scheme for PhD Studies in Science & Technology”.



M_{11}=HH+HV+VH+VV	M_{12}=HH+HV-VH-VV	M_{13}=PH+PV-MH-MV	M_{14}=RH+RV-LH-LV
M_{21}=HH-HV+VH-VV	M_{22}=HH-HV-VH+VV	M_{23}=PH-PV-MH+MV	M_{24}= RH-RV-LH+LV
M_{31}=HP-+HM+VP-VM	M_{32}=HP-HM-VP+VM	M_{33}=PP-PM-MP+MM	M_{34}= RP-RM-LP+LM
M_{41}=HR-HL+VR-VL	M_{42}=HR-HL-VR+VL	M_{43}=PR-PL-MR+ML	M_{44}= RR-RL-LR+LL

Tab.1 *Calculation of the 16-image Muelle matrix.*

The notation is as follows: the first term represents the input polarization state, while the second term represents the output polarization state. The states are defined as: H for horizontal, V for vertical, P for+45°, M for -45°, R for right circular, and L for left circular.

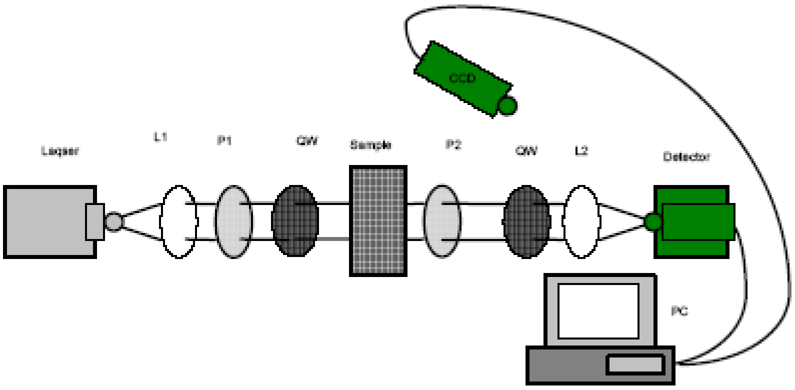
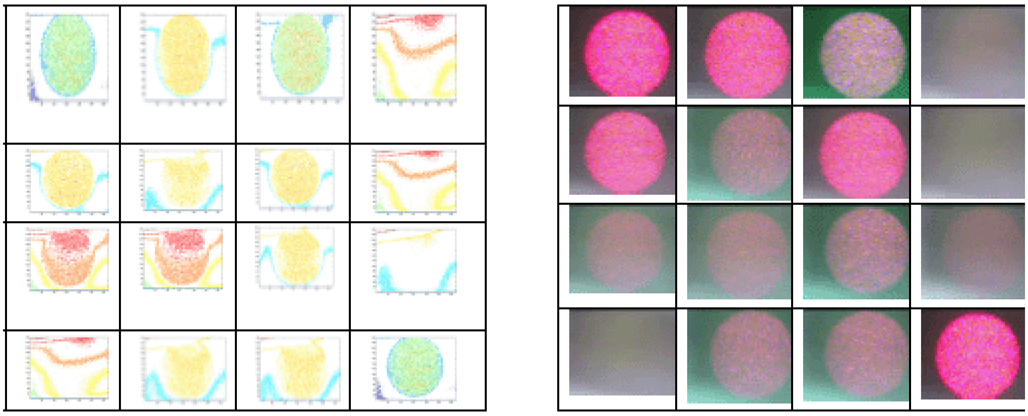


Fig.1 *Experimental setup for the measurement and imaging of transmitted Mueller matrix element.*

A He-Ne laser beam with an output power of 5 mW at a wavelength of 632.5 nm is used as the light source. The laser light is focused on polarizer P1 for obtaining linearly polarized light. The circularly polarized light is generated, by inserting a quarter mica retardation plate behind the linear polarizer. The out put polarized light is focus to polystyrene sphere suspension turbid medium, by lens L1 (f = 15 cm) and again pass through linear polarizer, quarter wave plate and recorded on photodiode detector and CCD camera, which is controlled and operated with Lab software.



(a) **(b)**

Fig.2 *(a)The two dimensional images of 16 Mueller matrix transmitted intensity elements, (b) the original shape of laser intensity profile by CCD camera for polystyrene sphere.*

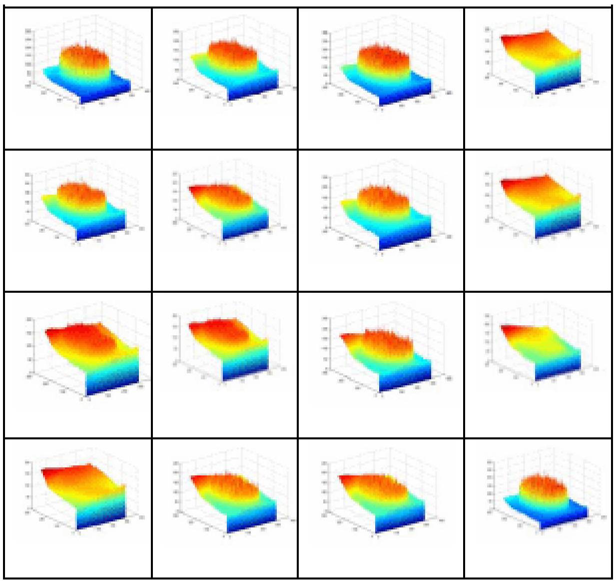


Fig.3 *The three dimensional images of 16 Mueller matrix transmitted intensity elements for polystyrene sphere.*

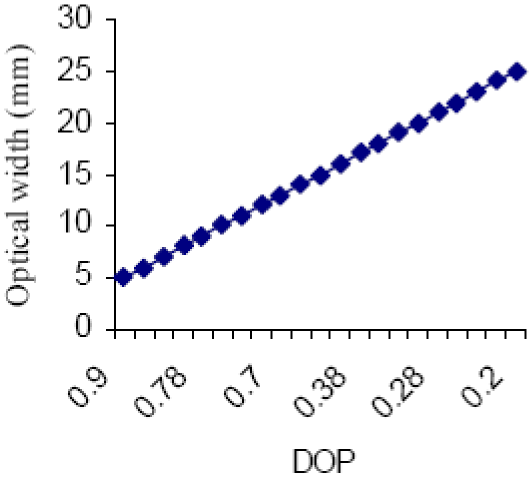


Fig.4 *Depolarizing horizontal linear Mueller matrix variation with respect to material width.*

1.021	0.567	-0.254	0.059
0.739	0.432	0.831	0.061
0.385	0.243	0.519	-0.213
0.0653	0.325	-0.136	0.983

Fig.5 *Mueller matrix data for transmitted polarized laser beam from polystyrene sphere suspension.*

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